Mark Scheme

Summer 2023

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 3B Further Statistics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Qu 1	Scheme	Marks	AO
(a)	[E(X) =] $-2\times0.25 + -1\times a + 0\times b + 1\times a + 3\times0.3$	M1	1.1b
	$= \underline{\mathbf{0.4}}$	A1	1.1b
		(2)	
(b)	$\mathrm{E}(X^2) = (-2)^2\times0.25 + (-1)^2\times a + 0 + 1^2\times a + 3^2\times0.3$ $(= 2a + 3.7)$	M1	2.1
	[Var(X) =] $3.9 = 2a + 3.7 - "0.4^2"$	dM1	1.1b
	$a = \underline{\mathbf{0.18}}$	A1	1.1b
	[Use of sum of probs = 1 implies $2a + b = 0.45$] $b = \underline{\mathbf{0.09}}$	A1ft	1.1b
		(4)	
(c)	$X_1 + X_2 > 3$ when $X_1 = 3, X_2 = 1$ $X_1 = 1, X_2 = 3$ $X_1 = 3, X_2 = 3$	M1	3.4
	[P $(X_1 + X_2 > 3)$=]		
	$"0.18"\times0.3 + 0.3\times"0.18" + 0.3\times0.3$ or $2\times0.3\times(0.3 + "0.18") - 0.3^2$	M1	1.1b
	$= \underline{\mathbf{0.198}}$	A1	1.1b
		(3)	
		(9 marks)	

	Notes
(a)	M1 for a correct attempt (at least 3 correct non-zero terms or products and addition) division by k $(k \neq 1)$ is M0 A1 for 0.4 o.e. (correct answer only scores 2 out of 2)
(b)	1st M1 for a correct attempt at E(X^2) (at least 3 correct non-zero products and addition) Missing brackets around –2 and –1 is M0 unless recovered 2nd dM1 (dep on 1st M1) for use of 3.9 = their $\mathrm{E}(X^2) - [\mathrm{E}(X)]^2$ ft their E(X) = 0.4 1st A1 for a = 0.18 o.e. 2nd A1 (dep on 1st M1 only) for b = 0.09 o.e. or their $b = 0.45 - 2\times"a"$ (provided both a and b are probabilities)
(c)	1st M1 for identifying at least 2 cases e.g. $X_1 = 3, X_2 \geqslant 1$ counts as 2 cases (ignore extras including any incorrect pairs identified) implied by at least two correct products of probs. or correct ft products of probs. 2nd M1 for a correct numerical expression for the probability ft their "0.18" A1 for 0.198 o.e.

Qu 2	Scheme	Marks	AO
(a)	$H_0 : \lambda = 1.7$ $\quad H_1 : \lambda \neq 1.7$	B1	2.5
	[X = no. of calls in 10 mins] $\quad X \sim$ Po(17)	M1	3.3
	$[P(X \geqslant 25) = 1 - P(X \leqslant 24)] = 0.0406463\ldots$ or CR: $X \geqslant 27$	A1	3.4
	[0.04..> 0.025/ 25 is not in CR so not significant] insufficient evidence of a change in rate of calls	A1	2.2b
		(4)	
(b)	[T = no. of calls longer than 8 minutes] $\quad T \sim$ B(70, 0.012)	M1	3.3
	[P($T > 2$) =] $P(T \geqslant 3) = 1 - P(T \leqslant 2) = 1 - 0.947725\ldots$	M1	3.4
	= awrt **0.0523**	A1	1.1b
		(3)	
(c)	[C = no. of calls out of 900 longer than 30 mins] [$C \sim$ B(900, p)] $\quad C \approx\sim$ Po(900p)	M1	3.3
	$P(C = 0) \approx e^{-900p} = 0.05$	M1	3.4
	$900p = -\ln(0.05)$ [= 2.9957…]	M1	1.1b
	$p = 0.003328\ldots$ awrt **0.00333**	A1	1.1b
		(4)	
		(11 marks)	

	Notes
(a)	B1 for both hypotheses correct which must be attached to H_0 and H_1 must be in terms of λ or μ allow either 1.7 or 17
	M1 for stating or using the correct Poisson model. may be implied by sight of awrt 0.0406/7 or awrt 0.959 or 0.9747… or better
	1st A1 for correct prob of awrt 0.04 or for correct CR found $X \geqslant 27$ ($X > 26$) (ignore lower tail CR if found) allow CV $X = 27$
	2nd A1 (dep on M1A1) for a correct conclusion in context mentioning "rate of calls" o.e. Allow e.g. 'The rate of calls is 1.7 per minute/17 per 10 minutes' Must be rate o.e. not "number" A0 if inconsistent comments are seen e.g. "reject H_0, no change in rate of calls"
(b)	1st M1 for sight or use of the correct binomial model. may be implied by sight of awrt: 0.0523 or 0.948 or 0.795 or 0.205
	2nd M1 for correct interpretation of more than 2 (allow 1 – 0.95 or better)
	A1 for awrt 0.0523 (correct answer only scores 3 out of 3)
SC:	Use of Po(70 ×0.012) leading to an answer of 0.0533(45…) and scores M1M1A0
(c)	1st M1 for sight or use of Po(900p) (as a suitable approx. to B(900, p)) (may be implied by correct answer awrt 0.00333)
	2nd M1 for a correct equation in p or correct use of P($C = 0$) from Po e.g. $e^{-\lambda} = 0.05$
	3rd M1 for a correct method to solve for p (allow $p = \pm \ln(0.05)/900$) or to solve for λ, i.e. $\lambda =$ awrt 3(.00)
	A1 for p = awrt 0.00333 **Must see Po used** condone $\frac{1}{300}$ o.e. Allow standard form (awrt 3.33×10^{-3}) or percentage (awrt 0.333%)
SC:	Use of Binomial gives 0.003323… awrt 0.00332 scores M0M0M0A1

Qu 3	Scheme	Mark	AO
(a)	$[X \sim B(5, 0.5)]$ $P(X = 0) = P(X = 5) = 0.03125$ <u>or</u> $P(X = 2)$ or $P(X = 3) = 0.3125$	M1	1.1b
	[multiply by 170 to get] $r =$ **<u>5.31</u>**(25) ; $s =$ **<u>53.1</u>**(25)	A1;A1 **(3)**	1.1b(x2)
(b)	H_0 : B(5, 0.5) is a suitable model H_1 : B(5, 0.5) is NOT a …	B1	2.5
	$\frac{(O_i-E_i)^2}{E_i}$: $\frac{(3-'5.31')^2}{'5.31'} = 1.00...$; $\frac{(10-26.56)^2}{26.56} = 10.3...$; $\frac{(45-'53.1')^2}{'53.1'} = 1.23...$; $\frac{(62-'53.1')^2}{'53.1'} = 1.48...$; $\frac{(38-26.56)^2}{26.56} = 4.92...$; $\frac{(12-'5.31')^2}{'5.31'} = 8.41...$	M1	1.1b
	$\frac{O_i^2}{E_i}$: $\frac{3^2}{'5.31'} = 1.69...$; $\frac{10^2}{26.56} = 3.76...$; $\frac{45^2}{'53.1'} = 38.1...$; $\frac{62^2}{'53.1'} = 72.3...$; $\frac{38^2}{26.56} = 54.3...$; $\frac{12^2}{'5.31'} = 27.1...$		
	$\sum \frac{(O_i - E_i)^2}{E_i}$ or $\sum \frac{O_i^2}{E_i} - 170 = 27.4$ … awrt **<u>27.4</u>** or awrt **<u>27.5</u>**	A1	1.1b
	Degrees of freedom is 6 – 1 = **<u>5</u>** , and critical value is **<u>11.07(0)</u>**	B1ft B1ft	1.1b(x2)
	[Significant result] <u>Marcus' model/B(5, 0.5)</u> is not a good fit. (o.e.)	A1 **(6)**	2.2b
(c)	$\hat{p} = \left[\frac{0\times3+1\times10+...+5\times12}{170\times5}\right] = 0.58588\ldots$ awrt **<u>0.586</u>**	B1 **(1)**	1.1b
(d)(i)	Need to pool (first 2) cells (0 and 1 since $E(0) < 5$) <u>and</u> use of $\hat{p}$	M1	2.4
	Degrees of freedom: 5 groups – 2 constraints = **<u>3</u>**	A1	1.1b
(ii)	Critical value is **<u>7.815</u>**	B1ft **(3)**	1.1b
(e)(i)	Nima's model is a good fit (since 1.62 < '7.815')/Marcus' is not <u>and</u> this suggests coin is biased/probability of head approx. 0.6	B1	2.4
(ii)	Nima's test suggests binomial is a good model <u>and</u> therefore independence of spins is a reasonable assumption	B1 **(2)**	2.2b **(15 marks)**

	Notes
(a)	M1 for 1 correct probability which may be embedded (0.03125 or 0.3125 or 0.5^5 or 5C2 $0.5^2 \times 0.5^3$)
	1st A1 for r = awrt 5.31 (condone $\frac{85}{16}$)
	2nd A1 for s = awrt 53.1 (condone $\frac{425}{8}$)
(b)	1st B1 for both hypotheses mentioning B(5, 0.5) or Marcus' distribution at least once
	M1 for at least one correct (ft) term or expression of the test statistic (accept 2sf)
	1st A1 for awrt 27.4 or awrt 27.5 (correct value here scores M1A1)
	2nd B1 for 5 <u>or</u> ft if 'their r' < 5, then df (= 4 – 1) = 3
	3rd B1 for 11.07(0) (or better) for ft df = 4 → 9.488 or df = 3 → 7.815
	A1 dep on 1st M1 for a suitable conclusion in context rejecting <u>B(5, 0.5)/Marcus' model</u> Must be compatible with their test statistic and their CV. Just 'Bin is not a good fit' is A0
	A0 if inconsistent comments are seen e.g. "do not reject H_0 , B(5, 0.5) is not a good fit"
(c)	B1 for awrt 0.586 allow $\frac{498}{850}$ o.e.
(d)(i)	M1 for both reasons, must mention pooling or show pooling or mention exp. value < 5 <u>and</u> use of estimated parameter
	A1 for df = 3 (must have scored the M1 for this mark)
(ii)	B1 for 7.815 (or better) allow this independent of the M1 only allow ft on df=4 → 9.488
(e)(i)	1st B1 for stating Nima's (binomial) model is a good fit/do not reject H_0 for Nima's model/Marcus' model is not a good fit <u>and</u> suggest that coin is probably biased/ $p > 0.5$ (p closer to 'their (c)')
	Only comparing 1.62 with '27.4' to reach $p > 0.5$ is incorrect and scores B0

(ii)	2nd B1 for mention of Nima's test suggests Binomial distribution is suitable and that spins are independent (ignore reference to Marcus' test for 2nd B1)

Qu 4	**Scheme**	**Mark**	**AO**
	[X = no. of rolls to 4 sixes] X~NegBin(4, $\frac{1}{6}$)	M1	3.3
	$\mu\left[=\frac{r}{p}\right]=$ **24** , $\sigma^2\left[=\frac{r(1-p)}{p^2}\right]=\frac{4\times\frac{5}{6}}{\frac{1}{36}}=$ **120**	A1, A1	1.1b(x2)
	$[\bar{X}\approx\sim]\mathrm{N}\left("24",\sqrt{\frac{"120"}{32}}^2\right)$	M1 M1	2.1,3.4
	P($\bar{X}<27.2$) = 0.95078… awrt **0.951**	A1	1.1b
		(6 marks)	

	Notes
	1st M1 for selecting the correct negative binomial model. May be implied by correct mean or variance NegBin on its own is M0 1st A1 for mean = 24 2nd A1 for variance = 120 $\sigma=120$ is A0 unless recovered 2nd M1 for writing or using of normal with mean 24 (may be implied by correct answer) ft their mean which may come from any distribution 3rd M1 for writing or using normal with standard deviation $=\sqrt{\frac{120}{32}}\left[=\sqrt{3.75}\right]$ ft $\frac{\text{their }\sigma}{\sqrt{32}}$ where σ may come from any distribution (may be implied by correct answer) 2nd A1 for awrt 0.951 (correct answer scores 6 out of 6)

Qu 5	Scheme	Marks	AO
(a)	$H_0 : \mu = 330$ $\quad H_1 : \mu < 330$	B1	2.5
	$[\bar{X} \sim]N\left(330, \left(\frac{8}{\sqrt{25}}\right)^2\right)$	M1	3.3
	$P(\bar{X} < C) = 0.05 \Rightarrow \frac{C-330}{8/\sqrt{25}} = -1.6449$	M1	3.4
	So $C = 327.368\ldots$ and critical region is: $\bar{X}$ < awrt **327**	A1	1.1b
		(4)	
(b)	$\bar{Y} \sim N\left(330, \left(\frac{8}{\sqrt{55}}\right)^2\right)$ and require $2 \times P(\bar{Y} < 328)$ (o.e.)	M1	3.3
	$= 0.063732\ldots$ awrt **0.0637**	A1	1.1b
		(2)	
(c)	$P(\bar{X} >$ "327.368…"$\mid \mu = 325)$ or $1 - P(\bar{X} <$ "327.368…"$\mid \mu = 325)$	M1	3.4
	$= 0.0694233\ldots$ awrt **0.0694**	A1	1.1b
		(2)	
		(8 marks)	

	Notes
(a)	B1 for both hypotheses in terms of μ
	1st M1 for stating or using the correct model – may be implied by use in later line. Condone X or any letter for $\bar{X}$
	2nd M1 for a correct equation for C Allow any z value that satisfies $1.6 < \lvert z \rvert < 1.7$ If standardisation equation not seen, this mark may be implied by CV = awrt 327 or CR: <awrt 327
	A1 for a correct CR allow just "< awrt 327" Condone e.g. $X < 327$ rather than $\bar{X} < 327$ Condone $\leqslant$
(b)	M1 for sight of correct model and attempt at $P(\bar{Y} < 328)$ (o.e.) Condone missing $2\times$
	A1 for awrt 0.0637 (correct answer scores 2 out of 2)
(c)	M1 for a correct (ft) statement may be implied by sight of e.g. $Z > \frac{"327.36..."-325}{8/5} = 1.48..$
	For $\mu = 325$ allow $\bar{X} \sim N(325, \ldots.)$
	Allow ft from a 2-tailed test in part (a)
	A1 for awrt 0.0694 (correct answer scores 2 out of 2)
SC	Sight of $P(328 < \bar{X} < 332 \mid \mu = 325)$ or $1 - P(\bar{X} < 328 \cup \bar{X} > 332 \mid \mu = 325)$ scores M1A0

Qu6	Scheme	Marks	AO
(a)	NegBin(r, p) has pgf $\left[\frac{pt}{1-(1-p)t}\right]^r$ and identify the connection	M1	2.1
	NegBin(2, $\frac{1}{3}$)	A1	2.2a
		(2)	
(b)	e.g. no. of rolls to achieve 5 or 6 (so that $p=\frac{1}{3}$) <u>twice</u> (oe)	B1ft	3.3
		(1)	
(c)(i)	$G'_X(t)=\frac{2t(3-2t)^2-(-2)\times 2(3-2t)t^2}{(3-2t)^4}$ <u>or</u> $\frac{6t}{(3-2t)^3}$	M1	2.1
		A1	1.1b
	$\underline{E(X)} = G'_X(1) =$ **<u>6</u>**	A1	1.1b
(ii)	$G''_X(t)=\frac{6(3-2t)^3-(-2)\times 3(3-2t)^2\times 6t}{(3-2t)^6}$ <u>or</u> $\frac{18+24t}{(3-2t)^4}$	M1	2.1
	$G''_X(1)=42$	A1	1.1b
	Var(X) = "42" + "6" − "6"^{2}	M1	1.1b
	$=$ **<u>12</u>**	A1	1.1b
		(7)	
(d)	$G_Y(t)=t^{10}\times\frac{1}{9}\left[1-\frac{2}{3}t^3\right]^{-2}=\frac{t^{10}}{9}\left[1+\ldots\frac{(-2)(-3)(-4)}{3!}\left(-\frac{2}{3}\right)^3 t^9\ldots\right]$	M1 A1	2.1 1.1b
	$P(Y=19) = \frac{32}{243}$	A1	1.1b
		(3)	
ALT	Identify that $Y=3X+4$	M1	
	($Y=19$ requires $X=5$ so) $P(X=5)=\binom{4}{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)$	A1	
		(13 marks)	

	Notes
(a)	M1 for identifying the NegBin distribution (allow NB for NegBin) A1 for $r=2$ and $p=\frac{1}{3}$
(b)	B1ft for identifying a suitable definition for X using a (fair) die, with $p=\frac{1}{3}$ and the second occurrence of the event, only ft their NegBin distribution in (a). A finite number of rolls is B0
(c)(i)	1st M1 for attempt to differentiate quotient or product. At least one uv′ style term correct. 1st A1 for a fully correct first derivative (needn't be simplified) 2nd A1 for E(X) = 6 NB this A1 depends on M1 only but M1A0A1 is possible
(ii)	2nd M1 for attempt to diff' quotient or product again. At least one uv′ style term correct. 3rd A1 for 42 (may be given for incorrect G″ provided their G″(1) gives 42 and M1 scored) Note all powers of $(3-2t)$ equal 1 when $t=1$ is substituted so can be used as a check 3rd M1 for correct use of pgf to find Var(X) 4th A1 dep on M3 for 12
(d)	M1 for writing pgf in suitable form to carry out binomial expansion 1st A1 for a correct expression for coefficient of t^{19} 2nd A1 for $\frac{32}{243}$ or exact equivalent

ALT	M1 for identifying connection $Y = 3X + 4$ 1st A1 for a correct numerical probability expression for P($X = 5$)		
Qu7	**Scheme**	**Marks**	**AO**
(a)(i)	X~Geo(0.2) or P($X = 4$) = $0.8^3 \times 0.2$	M1	3.3
	= **0.1024**	A1	1.1b
		(2)	
(ii)	T~NegBin(3,0.2) or P($T = 8$) = $\binom{7}{2}0.2^2 \times 0.8^5 \times 0.2$	M1	3.3
	= 0.05505… awrt **0.0551**	A1	1.1b
		(2)	
(iii)	F~B(10, 0.2) or P($F = 4$) = $\binom{10}{4}0.2^4 \times 0.8^6$	M1	3.3
	P($F = 4$) = 0.088080… awrt **0.0881**	A1	1.1b
		(2)	
(b)	P(R) = P($X \leqslant 4$) and X~Geo(0.2) | P($X \geqslant 1$), $X \sim$ B(4, 0.2)	M1	3.1b
	= 1 – P($X > 4$) = $1-0.8^4$ | = 1 – P($Y = 0$) = $1-0.8^4$	M1	3.4
	= 0.59(04)	A1	1.1b
	P(Y) = P($N \leqslant 7$) and N ~NegBin(3, 0.4)	M1	3.1b
	$0.4^3 + \binom{3}{2}0.4^3 0.6^1 + \binom{4}{2}0.4^3 0.6^2 + \binom{5}{2}0.4^3 0.6^3 + \binom{6}{2}0.4^3 0.6^4$ | $1-\left(\binom{7}{2}0.4^2 0.6^5 + \binom{7}{1}0.4^1 0.6^6 + \binom{7}{0}0.6^7\right)$	M1	3.4
ALT	P(Y) = P($W > 2$) where W~B(7, 0.4)	M1	
	= 1 – P($W \leqslant 2$) [= 1 – 0.419904]	M1	
	= 0.58(0096)	A1	1.1b
	R (has the greater probability)	A1	3.2b
		(7)	
		(13 marks)	

Notes

(a)(i) M1 for selecting the correct model. Stated or used which may be implied by ans.
A1 for 0.1024 or $\frac{64}{625}$ (accept 0.102) (correct answer scores 2 out of 2)

(ii) M1 for selecting the correct model. Stated or used which may be implied by ans.
Allow $0.2 \times$ P($V = 2$) from V ~B(7, 0.2)
A1 for awrt 0.0551 (correct answer scores 2 out of 2)

(iii) M1 for selecting the correct model. Stated or used may be implied by ans of 0.967(2)
A1 for awrt 0.0881 (correct answer scores 2 out of 2)

(b) 1st M1 for a correct distribution and prob. expression for P(R) (may be implied by 2nd M1)
2nd M1 for a correct numerical expression for P(R) (allow any equivalent expression)
1st A1 for awrt 0.590 or $\frac{369}{625}$ (accept 0.59 or better) awrt 0.590 implies M1M1A1
3rd M1 for a correct distribution and prob. expression for P(Y) (may be implied by 4th M1)
4th M1 for a correct numerical expression for P(Y) (allow any equivalent expression)
2nd A1 for awrt 0.580 or (accept 0.58 or better) awrt 0.580 implies M1M1A1

	3rd A1 dep on all other marks for *R* or correct description in words	Condone P(*R*) > P(*Y*)

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